# Domain Expansion: A Latent Space Construction Framework for Multi-Task Learning

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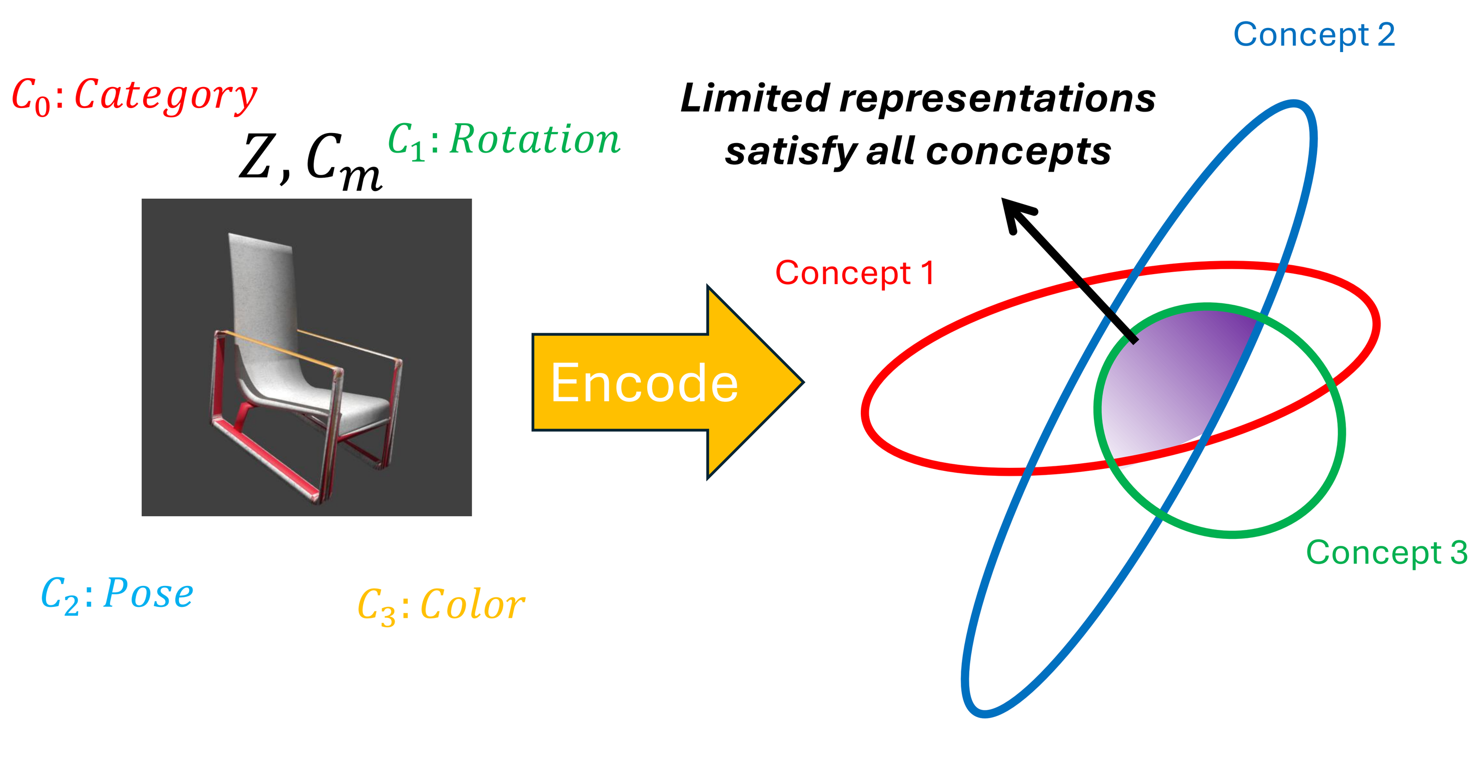
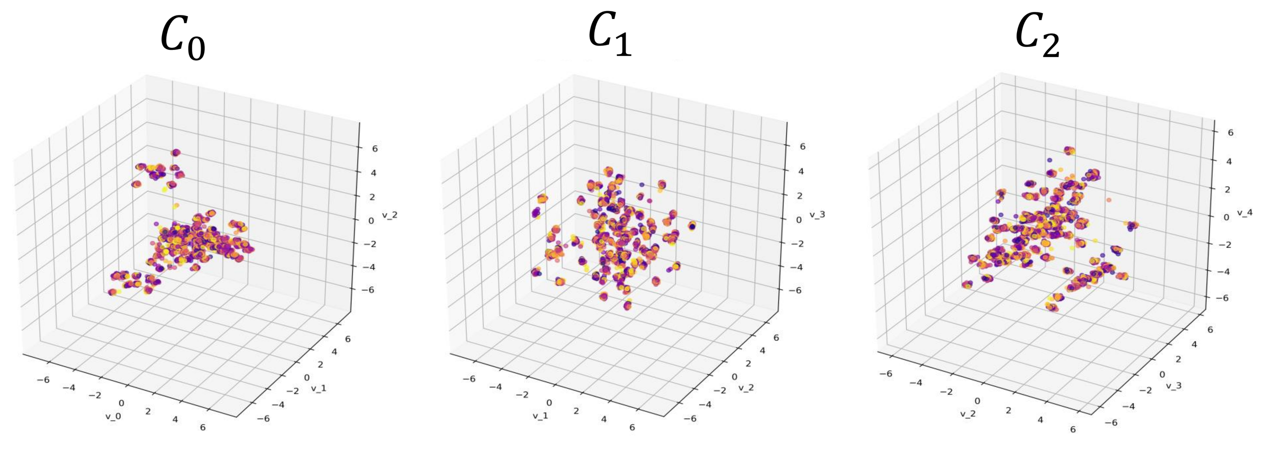
## Latent Representation Collapse

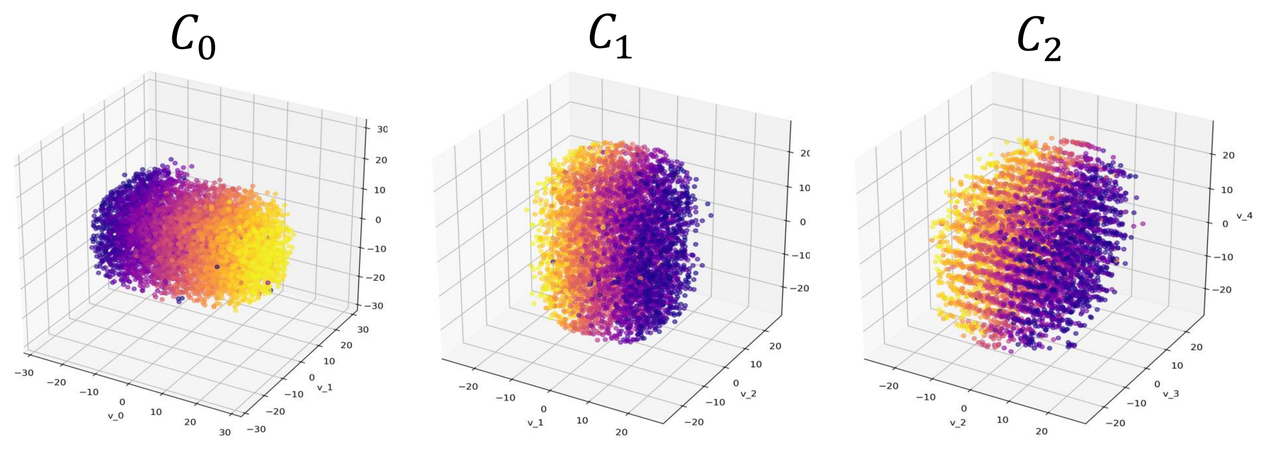
A visual input often contains rich concepts. If we train a single model to handle all concepts at the same time, the features will be pulled in different directions, leading to a shrunken latent space. This shrunken latent space often degrades the model's performance. We call this phenomenon **latent representation collapse**.

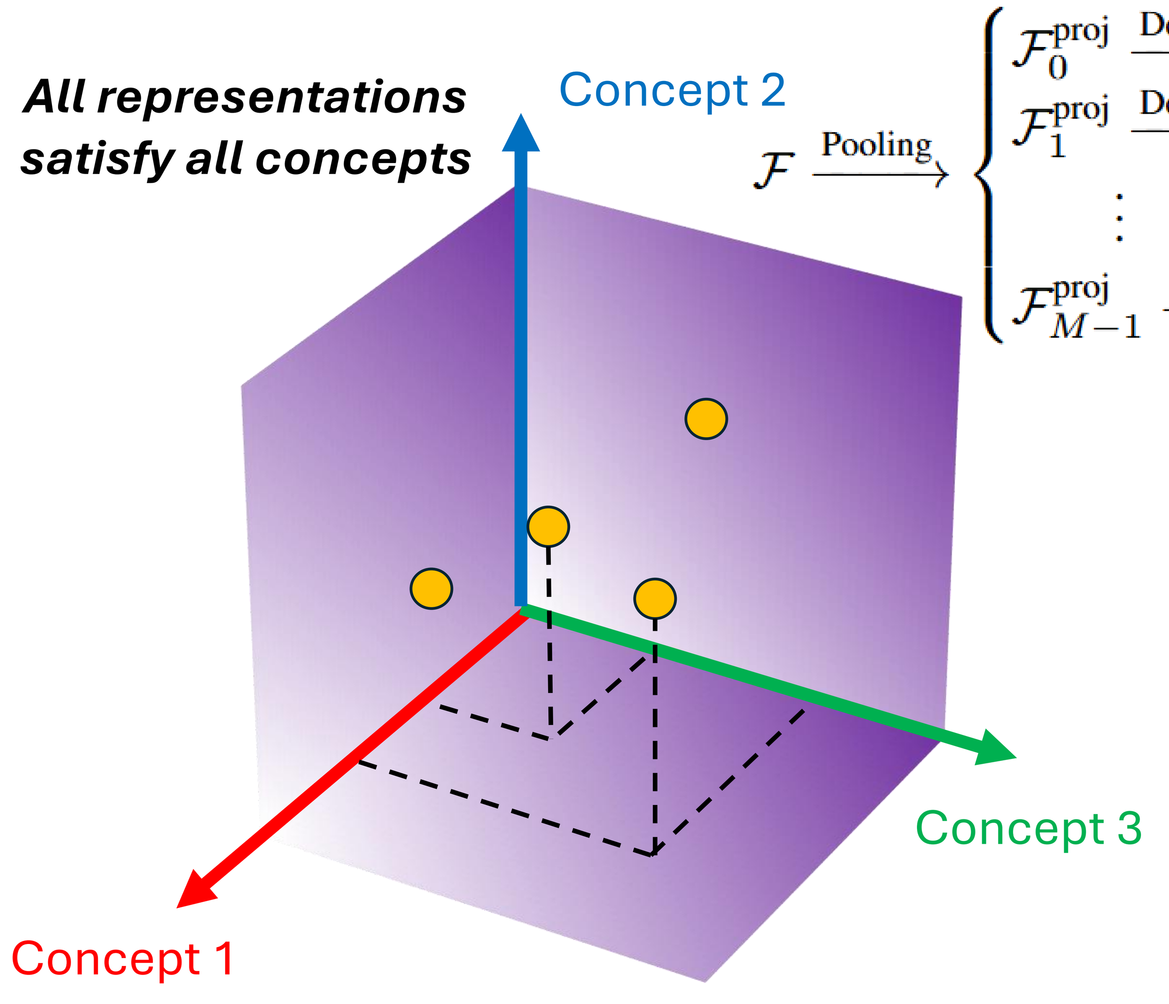



**Latent representation collapse**




**Domain Expansion (ours)**

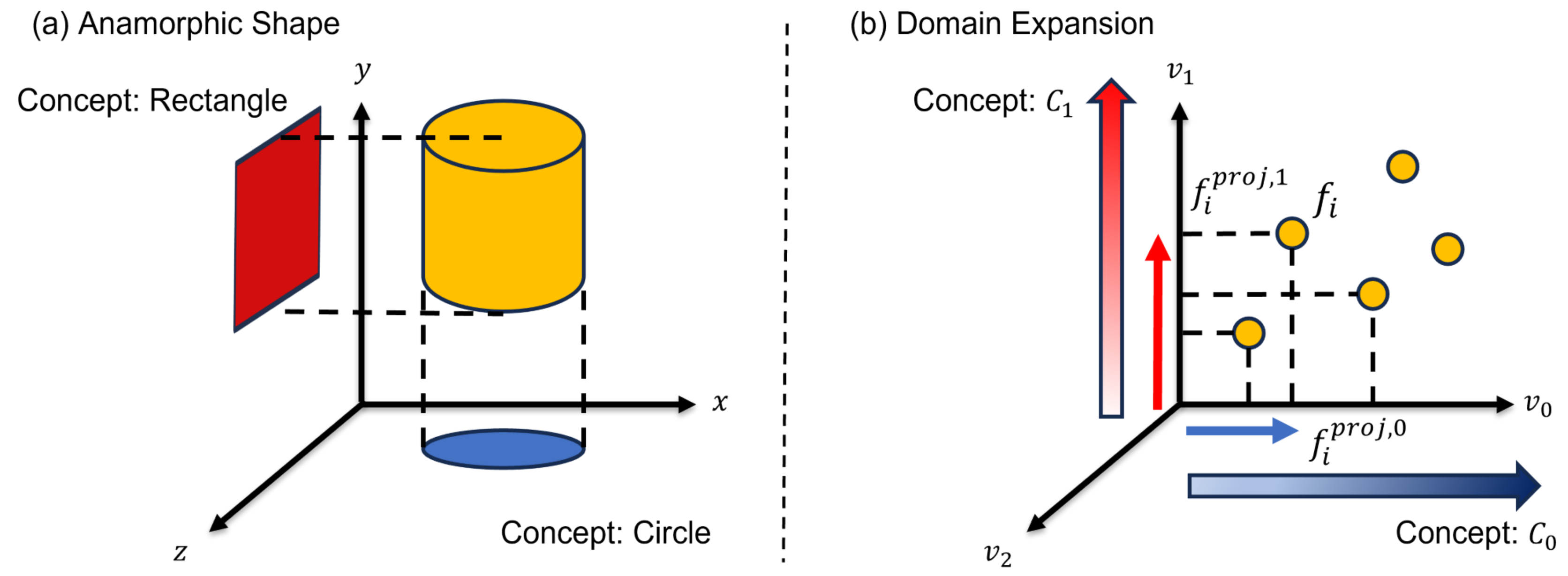


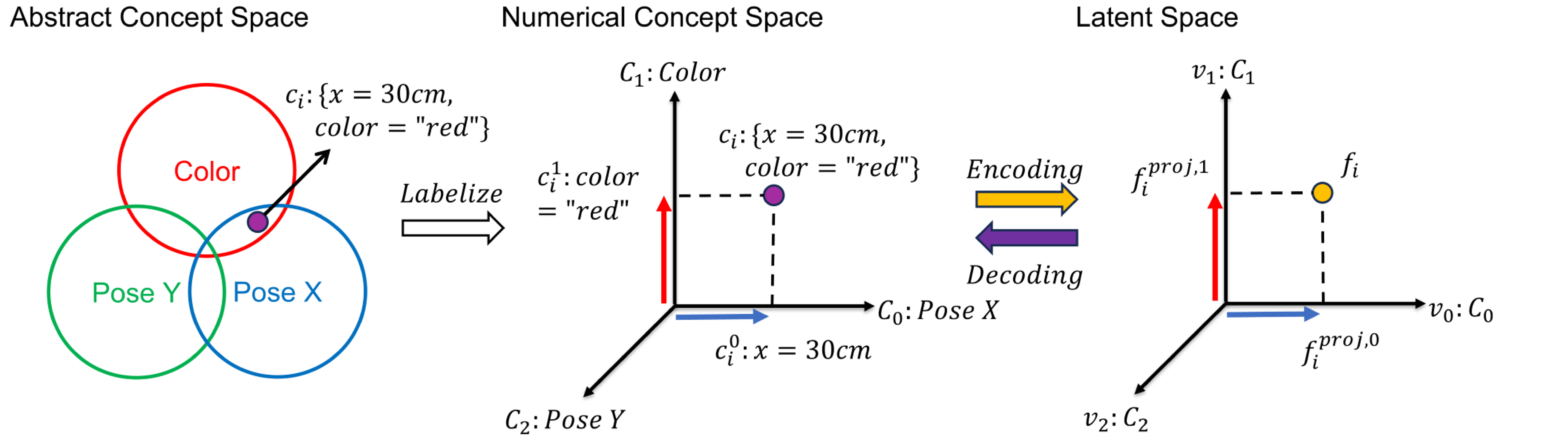




$$\mathcal{F} \xrightarrow{\text{Pooling}} \begin{cases} \mathcal{F}_0^{\text{proj}} \xrightarrow{\text{Dec}_0} \mathcal{C}_0 \\ \mathcal{F}_1^{\text{proj}} \xrightarrow{\text{Dec}_1} \mathcal{C}_1 \\ \vdots \\ \mathcal{F}_{M-1}^{\text{proj}} \xrightarrow{\text{Dec}_{M-1}} \mathcal{C}_{M-1} \end{cases}$$

## Orthogonal Pooling

In the latent space, we use eigenvectors to represent concepts. Each eigenvector is assigned to a single target concept. This assignment ensures that the 1D subspace spanned by **the eigenvector is dedicated exclusively to each concept.**




## Properties and Operators

- ***Property 1: Multi-concept Encoding.***

$$f_i \xrightarrow{\text{Pooling}} \{f_i^{\text{proj},0}, \ldots, f_i^{\text{proj},M-1}\} \xrightarrow{\text{Dec}} \{c_i^0, \ldots, c_i^{M-1}\} \rightarrow c_i.$$

- ***Property 2: Orthogonality of Target Concepts.***

$$\mathcal{F}_0^{\text{proj}} \perp \mathcal{F}_1^{\text{proj}} \perp \cdots \perp \mathcal{F}_{M-1}^{\text{proj}} \implies \mathcal{C}_0 \perp \mathcal{C}_1 \perp \cdots \perp \mathcal{C}_{M-1}.$$

- ***Operator 1: Concept-Specific Adjustment*** **($\oplus^{\text{m}}$)** ***and*** **($\ominus^{\text{m}}$).**

$$c_i \oplus^m c_\Delta^m \rightarrow \{c_i^0, \ldots, \{c_i^m \oplus^m c_\Delta^m\}, \ldots c_i^{M-1}\}$$
$$\xrightarrow{\text{Dec}^{-1}} \{f_i^{\text{proj},0}, \ldots, \{f_i^{\text{proj},m} + f_\Delta^{\text{proj},m}\}, \ldots, f_i^{\text{proj},M-1}\}$$
$$\xrightarrow{\text{Reconst}} f_i + f_\Delta^{\text{proj},m}.$$

- ***Operator 2: Concept Composition*** **($\oplus$)** ***and*** **($\ominus$).**

$$c_p \oplus c_q \rightarrow \{\{c_p^0 \oplus^0 c_q^0\}, \{c_p^1 \oplus^1 c_q^1\}, \ldots, \{c_p^{M-1} \oplus^{M-1} c_q^{M-1}\}\}$$
$$\xrightarrow{Dec^{-1}} \{\{f_p^{proj,0} + f_q^{proj,0}\}, \{f_p^{proj,1} + f_q^{proj,1}\}, \ldots, \{f_p^{proj,M-1} + f_q^{proj,M-1}\}\}$$
$$\xrightarrow{\text{Reconst}} f_p + f_q.$$

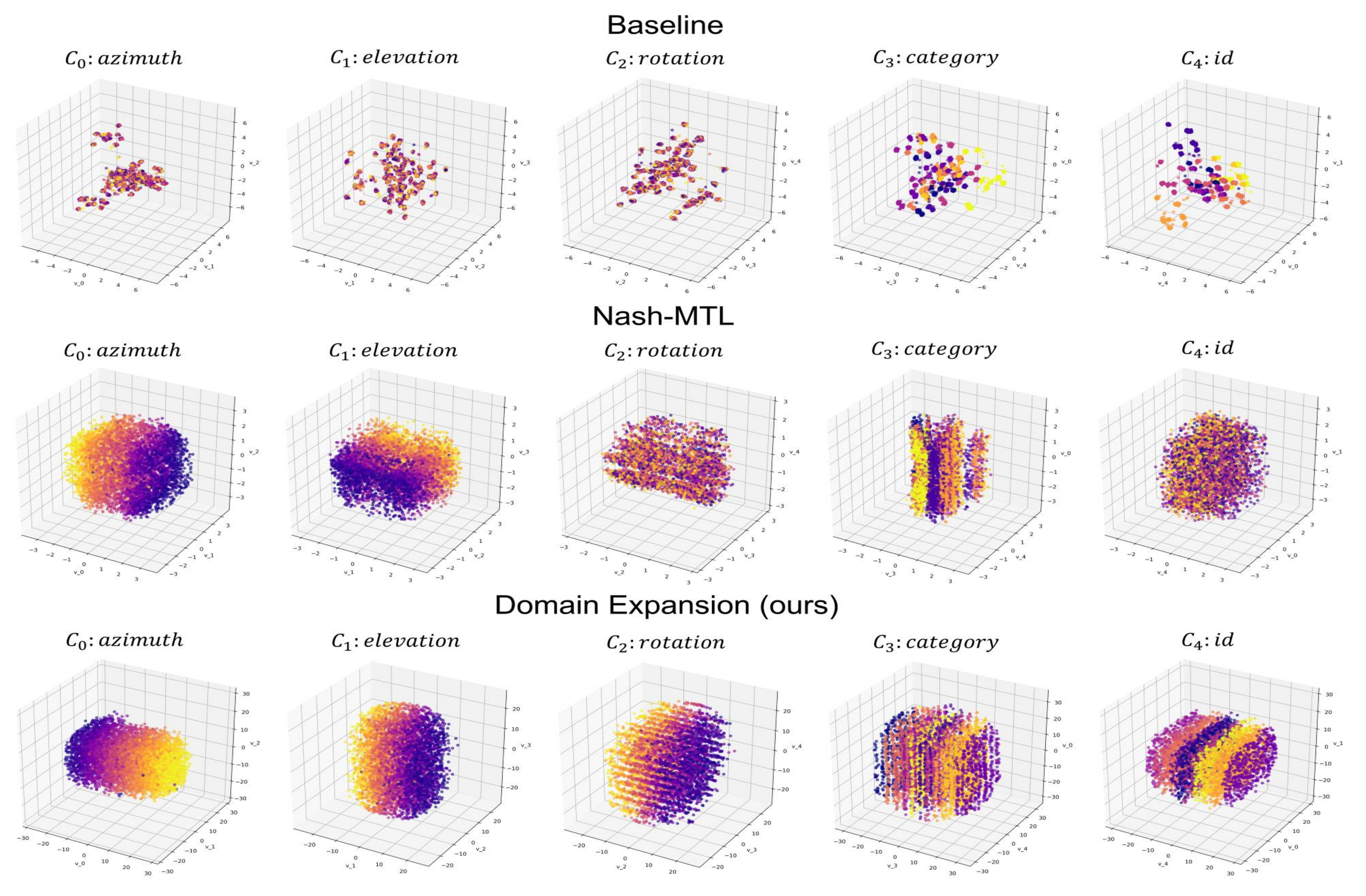



Table 1: Comprehensive comparison of representation quality, predictive performance, and concept composition. Arrows indicate whether higher (↑) or lower (↓) values are better.

| Objective Set | Method | Representation & Predictive Performance | | | | | | | | | | Concept Comp. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Spearman ↑ | | | V-score ↑ | | MAE° ↓ | | | Acc. ↑ | | Sim. ↑ |
| | | az | el | rot | cat | id | az | el | rot | cat | id | ⊕ and ⊖ |
| Objective Set 1 | baseline | 0.41 | 0.34 | 0.35 | 0.16 | 0.14 | 0.12 | 0.09 | 0.09 | 0.28 | 0.37 | 0.22 |
| | FAMO | 0.49 | 0.41 | 0.42 | 0.00 | 0.00 | 0.12 | 0.09 | 0.09 | 0.19 | 0.18 | 0.28 |
| | Nash-MTL | 0.38 | 0.41 | 0.42 | 0.00 | 0.00 | 0.11 | 0.09 | 0.09 | 0.17 | 0.13 | 0.28 |
| | IMTL | 0.31 | 0.16 | 0.16 | 0.39 | 0.28 | 0.14 | 0.11 | 0.12 | 0.92 | 0.79 | 0.14 |
| | **Ours** | 0.95 | 0.87 | 0.85 | 0.99 | 0.91 | 0.08 | 0.08 | 0.09 | 0.99 | 0.97 | 0.95 |
| Objective Set 2 | baseline | 0.01 | 0.01 | 0.01 | 0.99 | 0.00 | 0.77 | 0.38 | 0.38 | 0.99 | 0.99 | 0.42 |
| | FAMO | 0.28 | 0.23 | 0.22 | 0.99 | 0.00 | 0.19 | 0.14 | 0.13 | 0.99 | 0.99 | 0.28 |
| | Nash-MTL | 0.45 | 0.39 | 0.39 | 0.15 | 0.00 | 0.12 | 0.08 | 0.09 | 0.99 | 0.99 | 0.35 |
| | IMTL | 0.39 | 0.18 | 0.16 | 0.99 | 0.00 | 0.15 | 0.11 | 0.13 | 0.99 | 0.99 | 0.28 |
| | **Ours** | 0.95 | 0.87 | 0.85 | 0.98 | 0.96 | 0.07 | 0.08 | 0.09 | 0.98 | 0.94 | 0.93 |